Hamiltonians for Light-Matter Interaction

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The aim of this seminar is to establish, beyond any doubt, the correctness of the Power-Zienau-Woolley (PZW) Hamiltonian of quantum electrodynamics (QED) [1].

We begin with the classical Lagrangian for the interaction of a collection of charged particles with the electromagnetic field, and proceed via Hamilton's principle of least action, the calculus of variations, and the canonical quantisation scheme to obtain the gauge-invariant minimal-coupling Hamiltonian [2,3]. It is then shown how a PZW transformation [4,5] yields an equivalent Hamiltonian for the total radiation-matter system in which atoms and molecules couple to Maxwell fields through their polarisation, magnetisation and diamagnetisation distributions. Furthermore, all static inter-particle interactions are completely eliminated by this unitary transformation.

The often-deployed multipolar Hamiltonian is shown to arise in the Coulomb gauge when the path for each term in the sum over charged particles of the electric polarisation field is taken to be a straight line from some origin to the position of the particle. Alternative routes to the multipolar Hamiltonian through the generation of equivalent Lagrangians [6] or by effecting a gauge transformation [7] are also presented.

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