# Many-Body Quantum Interference on Hypercubes

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Quantum walks on hypercube (HC) graphs [1–3] attracted much attention owing to their potential applicability to quantum simulation as well as the promise of robust exponential speed-ups compared to classical random walks in search algorithms [1–3]. Beyond the regime of distinguishable particles, however, one has to take many-body quantum interferences into account, which give rise to intricate evolution scenarios. In case of HC-graphs, symmetries of the unitary scattering matrix alleviate the complexity and even allow the analytic formulation of suppression laws, which predict final states, with a probability vanishing due to total destructive interference. To date, only few such symmetric unitaries, like the discrete Fourier-transformation [4] or Sylvester matrices [5], have been investigated, leaving a general lack of knowledge about many-body evolution in higher-dimensional systems.
Here, we study many-body destructive interferences emerging in hypercubes of dimension $d$, generalized to have the same but arbitrary substructure on all vertices. Such graphs are described by unitaries of the form

$$\hat{\mathcal{U}} = \frac{1}{\sqrt{2^d}}\,\hat{U} \otimes \begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}^{\otimes d}. \tag{1}$$

Each node of the hypercube is represented by an $m \times m$ subunitary $\hat{U}$, as visualized in figure 1. We find that initial many-particle states with even particle number, being invariant under certain symmetry operations, show a large quantity of suppressed final states with the condition for suppression determined solely by the symmetry of the input state.

In conclusion, we show, that symmetries in many-particle scattering scenarios cause conceptional simplifications and allow the definite exclusion of certain outcomes. Our findings reveal new insights in particle statistics for ensembles of indistinguishable bosons and fermions and may be a first step towards many-particle quantum protocols in higher-dimensional structures.

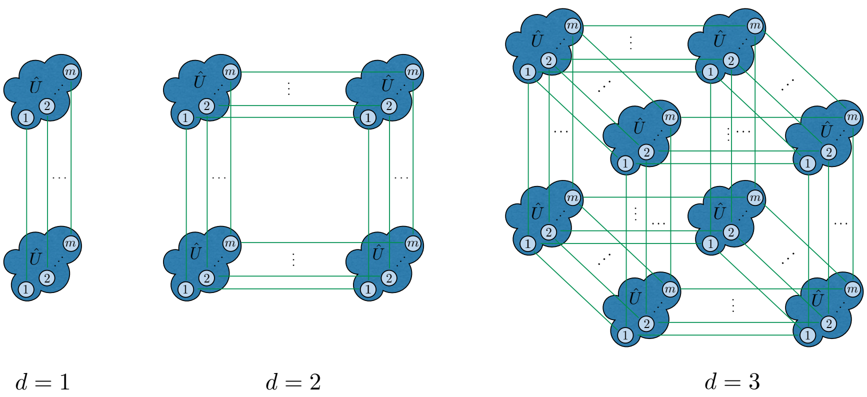


FIG. 1: Composition of graph structures under consideration. Arbitrary, but identical sub-lattices, composed of $m$ modes and visualized by blue clouds, are arranged in a hypercube structure of arbitrary dimension $d$. For simplicity, only possible structures up to $d = 3$ are shown.

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